EXHIBIT 3

TEACHING GEOMETRY WITH MAGNET SPHERE KITS

DAVID A. RICHTER

ABSTRACT. The purpose of this note is to summarize potential uses for small rare-earth magnet sphere kits, commonly sold under the brand names Buckyballs, Zen Magnets, Nanodots, and so on, as a tool in teaching geometry in college-level courses.

1. INTRODUCTION

The value of magnet spheres in collegiate mathematics education is difficult to ascertain. In the two years that I have had access to a significant number of them, I have not yet had the opportunity to use magnet sphere kits in a classroom setting. Also, since they have not been available in the public domain for more than a few years, few of my colleagues have had similar opportunities. From meetings with students in my office and from working with them on my own, I have conceived of some potential for a greater role in teaching certain concepts in core mathematics courses. Throughtout my career, I have used a variety of manipulatives in teaching various classes that involve concepts in geometry. As one might expect, the market for geometry manipulatives is *very* crowded. The problem here is to explain the particular role that magnet sphere kits may have in a realm that is already nearly saturated. Summarizing, magnet spheres provide a means for quickly crafting some natural geometric objects that might otherwise require more time than is typically allowed in a single class meeting.

Note. This document displays several free-view stereograms, with the cross-eyed pair on the left and the parallel-view pair on the right. One of the themes that I would like to convey here is that studying mathematics, especially geometry, often results in the creation of beautiful shapes, and so this is a way to excite students' minds. Thus, these photos are intended to showcase some of the more interesting models one might build in a classroom setting while a student is trying to acquire the central concepts in the curriculum.

2. SPECIFIC PEDAGOGY

The courses that I teach (on a rotating basis) where I see a potential use for magnet spheres are Introduction to Geometry (Math 3400 at Western Michigan University) and Modern Algebra (Math 3300 at WMU). Introduction to Geometry is intended as a refresher of Euclidean geometry and an introduction to more abstract geometric

Date: July 20, 2014.

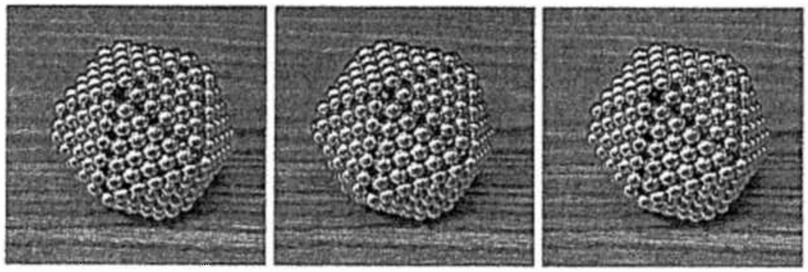

FIGURE 1. The Regular Icosahedron, 300 spheres

concepts for future high-school teachers. Modern Algebra is a traditional course in groups, rings, fields, homomorphisms, etc., intended for all of our students majoring in mathematics. Introduction to Geometry can be somewhat eclectic, so I imagine flexibility in creating instructional "modules" around the use of magnet spheres. Such modules, each using a number from twelve to several hundred magnets and some supplementary materials, would facilitate the acquisition of standard concepts in Euclidean geometry, especially space forms (including polyhedra), convexity, and symmetry. A study of symmetry groups is also a possible topic in Modern Algebra, although the curriculum for this course is more rigid than that of Introduction to Geometry. Due to the structure of the magnet sphere kits and the ideas that they are intended to convey, these modules would have to be intensely focused, and it would be a challenge to do this in a classroom with more than about 15-20 students. For most of these modules, I would divide the class into teams of 2-4 students per team and have them work the exercises during class.

Classical Solids. Magnet sphere kits provide a novel and elegant way to build models of the Platonic solids, Archimedean solids, and some variants. The smallest model that one can make with icosahedral symmetry uses 60 magnets each centered at the vertices of a small rhombitruncated icosidodecahedron. The simplest way to make this is by forming 12 pentagonal rings with 5 magnets each, and then attaching them as if they were forming a regular dodecahedron, making certain that they all have the same orientation with respect to their magnetic poles. One may make larger models using the same principles, building them by first crafting "panels" in the shapes of triangles, squares, pentagons, and so on. For example, a regular icosahedron can be crafted with precisely 300 magnets, as in Figure 1. In an advanced class, it may be required that the students build models of the Platonic or related solids. This is often done with paper, but it is time-consuming and cumbersome. Magnet spheres would allow a similar exercise while using less time assembling the pieces. For example, it is not hard to learn to make the 60-magnet icosahedral ball in less than 5 minutes.

Duality and Euler's formula. Duality is a fundamental concept in 3-dimensional Euclidean geometry, as is Euler's formula

$$V - E + F = 2,$$

which relates the numbers of vertices, edges, and faces of a convex polyhedron. Since these ideas are so closely related, they are often studied together in Introduction to

FIGURE 2. Dual Polyhedra

Geometry. A typical exercise for a student is to study some polyhedron models and see how duality and Euler's formula might work. One avenue for this might be to use magnet spheres to build a family of polyhedra which might be called "generalized deltahedra"; these are polyhedra whose faces are all equilateral triangles (shaped like the Greek letter Δ). It is a routine exercise to prove that there are only 8 convex deltahedra, due to the constaint that not more than 5 triangles share a vertex. Magnet spheres allow the construction of a considerably larger family where 6 or 7 triangles are allowed to share a vertex. For each of these, one may also make a dual model using magnet spheres. One of these dual pairs is pictured in 2. This particular polyhedron, resembling a rounded regular octahedron, uses precisely 24 triangles with 9 magnet balls per triangle. Appearing in the same picture is its dual, built using 8 hexagonal panels and 6 square panels. (It's notable that this polyhedron uses precisely one Zen Magnets kit!) Building these models, one inevitably appreciates both the tremendous constraint expressed by Euler's formula and also the fact that duality is a type of geometric isomorphism. There are scores of similar dual pairs in this family, and it would be instructive to guide a student in discovering them on his/her own.

Symmetries in Three Dimensions. One of the charms of using magnet spheres is that, with a little practice, one can build various elegant shapes with unusual symmetry groups. Thus, a possible exercise here is for the student to build a small collection of magnet sphere models (generalized deltahedra, for example) and then sort them them according to their symmetry groups. A converse to this exercise, with essentially the same lesson, would be to see how many different forms one can make which have a prescribed symmetry group. Magnet spheres offer the advantage that a student could work this relatively efficiently in a classroom under the supervision of his/her instructor.

Convexity and Curvature. A solid Ω is "convex" if for every pair of points in Ω, the entire line segment joining them also lies in Ω. Curvature is a local measure of whether a surface looks round like a ball or a saddle. The concepts are related, for one may verify, for example, that a surface which has a saddle point cannot bound a convex region. One may use magnet spheres to see this, again using generalized deltahedra as described above. Thus, after a student becomes skilled at making pentagonal and hexagonal panels, one can then teach a lesson about curvature when a student is successful in making a heptagonal or an octagonal panel.

FIGURE 3. A Möbius Band

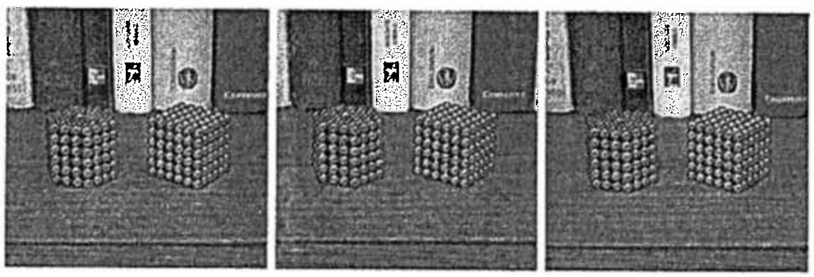

FIGURE 4. Hexagonal Prism and Cube

The Möbius Band. This is a popular and intriguing shape because it is a locally 2-dimensional object which seems to have only one "side". It is a traditional exercise to craft a Möbius band using paper, but one may make a satisfactory model using magnet spheres. The challenge for the student is to try to figure out how to make a model using magnet spheres after s/he has seen it made from paper. Such an exercise would instruct the student on the concept of the Möbius band, but also give her/him a slightly deeper understanding of the geometry of magnets.

Crystalline Forms. The densest way to pack a small set of magnet spheres is probably into a hexagonal prism, as in the way a set of 216 Zen Magnets are often packaged. Buckyballs were sold as cubes with 5 or 6 magnets to each side. These forms appear in Figure 4. If I were to use magnet spheres in a classroom setting, then learning to build these forms would be the first exercises they would master. Not only would a student learn about how some crystals form and how their symmetry groups act, but also they would learn an ideal way to account for all of the magnets. If someone cannot complete a hexagonal prism, consisting of 6 layers with 36 magnets each, then s/he should immediately retrieve the missing magnet(s).

Regular Skew Polyhedra. This is an advanced exercise one can perform if one has at least a few hundred magnets and the knowledge of how to build some polyhedra using hexagonal panels. A regular skew polyhedron shares some attributes with the Platonic solids. To be specific, every facet of a regular skew polyhedron is a regular polygon identical to every other facet, and the arrangements of facets at every

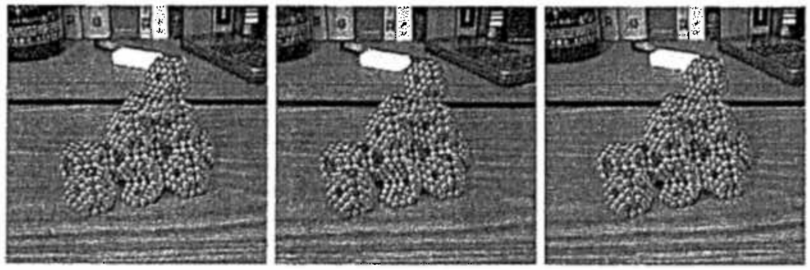

FIGURE 5. A Regular Skew Polyhedron

FIGURE 6. A Regular Skew Polyhedron

vertex is identical to every other such arrangement. However, for a regular skew polyhedron, we assume that the facets at each vertex alternate direction, hence the term "skew". They also differ in that they are infinite polyhedra, corresponding roughly to tilings of 3-dimensional space by identical convex solids. There are two that one may readily build with magnet spheres, provided the student knows how to make hexagonal panels and understands some of the structures of Archimedean solids. Figures 5 and 6 show a couple local realizations these two forms. Building these from magnet spheres, the student would gain a better appreciation for packings, space forms, and crystalline forms other than the hexagonal prisms and cubes described above.

3. CONCLUSION

We don't yet know the value of magnet spheres in collegiate mathematics education. I am not aware of anyone even attempting to use magnet spheres for teaching the specific concepts described above. So far, magnet spheres have been marketed primarily as a sculpting medium, a stress-reliever, or as a toy for adults. However, these are all functions of the brevity that magnet sphere kits have been on the market. Also, using magnet sphere kits for studying pure mathematics is relatively marginal compared to these; given the availability of other competing manipulatives for teaching geometry, it is not surprising that many of my colleagues are not even aware that magnet spheres might be used in this way. I don't envision ever requiring students to purchase magnet sphere kits in the same way that we require them to buy textbooks and graphing calculators. Rather, as I described above, I envision making small, portable modules that I can bring to the classroom during the moments when it fits into the curriculum. Obviously I am very enthused about working with magnet sphere kits in this way. Thus, as soon as I am back in our department's rotation for teaching either Introduction to Geometry or Modern Algebra, I will craft instruction of a few concepts with the use of magnet sphere kits and gain a better understanding of their value.

References

[1] H. S. M. Coxeter. *Twelve Geometric Essays.* Southern Illinois University Press, Carbondale, Illinois, 1968.

[2] H. S. M. Coxeter. *Introduction to Geometry.* 2nd ed. John Wiley & Sons, New York, 1969.

[3] H. S. M. Coxeter. *Regular Polytopes,* 3rd ed. Dover Publications, New York, 1973.

[4] Branko Grünbaum. *Convex Polytopes.* Interscience, London 1967;

[5] Günter M Ziegler. *Lectures on Polytopes.* Springer-Verlag, New York, 1995.

Department of Mathematics, Western Michigan University, Kalamazoo MI 49008-5248, 269-387-4555, david.richter@wmich.edu